LECTURE-2

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NO-1-TO-4

Solve the following differential equations

2. $y' = \sin 3x$

$\Rightarrow \frac{dy}{dx} = \sin 3x$

$\Rightarrow dy = \sin 3x\, dx$

Integrating both sides,

$\int dy = \int \sin 3x\, dx$

$\Rightarrow y = -\frac{1}{3}\cos 3x + C_1$

3. $y'' = x^{-4}$

$\Rightarrow \frac{d^2y}{dx^2} = x^{-4}$

$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = x^{-4}$

$\Rightarrow d\left(\frac{dy}{dx}\right) = x^{-4}\, dx$

Integrating both sides,

$\int d\left(\frac{dy}{dx}\right) = \int x^{-4}\, dx$

$\Rightarrow \frac{dy}{dx} = \frac{x^{-3}}{-3} + C_1$

$\Rightarrow dy = \left(-\frac{1}{3}x^{-3} + C_1\right) dx$

Again integrating both sides,

$\int dy = \int \left(-\frac{1}{3}x^{-3} + C_1\right) dx$

$\Rightarrow y = -\frac{1}{3}\,\frac{x^{-2}}{-2} + C_1 x + C_2$

$\Rightarrow y = \frac{1}{6}x^{-2} + C_1 x + C_2$.

NO-5-TO-9

State the order of the diffrential equation and verify that the given function is a solution.

6. $y'' + y = 0$ —— (1) $\quad y = a\cos x + b\sin x$ —— (2)

Ans:- order of the given DE is 2

Now $y = a\cos x + b\sin x$

$y' = -a\sin x + b\cos x$

$y'' = -a\cos x - b\sin x$

Putting in equation (1)

LHS:- $y'' + y$

$= -a\cos x - b\sin x + a\cos x + b\sin x$

$= 0 = RHS.$

Thus equation (2) is a solution of equation (1)

9. $x + yy' = 0$ —— (1) $\quad x^2 + y^2 = 1$ —— (2)

order of the given DE is 1.

Diffrentiating equation (2) both sides w.r. to x

$x^2 + y^2 = 1$

$\Rightarrow 2x + 2yy' = 0$

$\Rightarrow x + yy' = 0$ which is the required diffrential equation (1)

Thus equation (2) is a solution of equation (1)

NO-12-TO-17

Verify that y is a solution of the given DE. Determine the value of c using the initial condition.

14. $xy' = 3y$ —— (1) $\quad y = cx^3$ —— (2)

$y = 16$ when $x = -4$ i.e $y(-4) = 16$

Diff. equation (2)

$$y = cx^3$$

$$y' = 3cx^2$$

LHS $\quad xy' - 3y$

$= x(3cx^2) - 3(cx^3)$

$= 3cx^3 - 3cx^3$

$= 0 = RHS$

Thus equation (2) is a solution of equation (1)

Now $y = 16$ when $x = -4$

Putting in equation (2)

$$16 = c(-4)^3$$

$$\Rightarrow 16 = -64c \Rightarrow c = -\frac{1}{4}$$

Putting the value of c in equation (2)

$$\boxed{y = -\frac{1}{4}x^3}$$

There are several methods for solving First order diffrential equations. Some of the methods are

1. Variable separable
2. Method of substitution
3. Homogeneous DE
4. Exact DE
5. Integrating Factor
6. Linear DE
7. Bernoulli's DE.

Method of variable separable

$$\text{Let } \frac{dy}{dx} = \frac{M(x)}{N(y)}$$

$$\Rightarrow N(y)\,dy = M(x)\,dx$$

Integrating both sides,

$$\boxed{\int N(y)\,dy = \int M(x)\,dx + C}$$

That means take the y variable to dy and x variable to dx, integrating both sides we can get the result.

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Solve the following DE.

6. $y' = -ky^2$

$$\Rightarrow \frac{dy}{dx} = -ky^2$$

$$\Rightarrow \frac{dy}{y^2} = -k\,dx$$

Integrating both sides,

$$\int \frac{dy}{y^2} = \int -k\,dx$$

$$\Rightarrow -\frac{1}{y} = -kx + C_1.$$

7. $xy' = y^2 + y$

$$\Rightarrow x\frac{dy}{dx} = y(y+1)$$

$$\Rightarrow \frac{dy}{y(y+1)} = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{y} - \frac{1}{y+1}\right) dy = \frac{dx}{x}$$

Integrating both sides,

$$\int \left(\frac{1}{y} - \frac{1}{y+1}\right) dy = \int \frac{dx}{x}$$

$$\Rightarrow \ln|y| - \ln|y+1| = \ln|x| + \ln c$$

$$\Rightarrow \ln\left|\frac{y}{y+1}\right| = \ln|xc|$$

$$\Rightarrow \frac{y}{y+1} = xc$$

12-TO-19

Solve the following initial value problems

13. $xy' + y = 0 \qquad y(2) = -2$

$$\Rightarrow x\frac{dy}{dx} + y = 0$$

$$\Rightarrow x\frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides,

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$\Rightarrow \ln|y| = -\ln|x| + \ln c$

$\Rightarrow \ln|y| = \ln\left|\frac{c}{x}\right|$

$\Rightarrow y = \frac{c}{x}$ ——— (1)

$y(2) = -2$ i.e $y = -2$ when $x = 2$.

Putting in equation (1)

$$-2 = \frac{c}{2} \Rightarrow \boxed{c = -4}$$

Putting the value of c in equation (1)

$$\boxed{y = -\frac{4}{x}}$$

19. $L\frac{dI}{dt} + RI = 0 \qquad I(0) = I_0$

$\Rightarrow L\frac{dI}{dt} = -RI$

$\Rightarrow \frac{dI}{I} = -\frac{R}{L}dt$

Integrating both sides.

$$\int \frac{dI}{I} = -\frac{R}{L}\int dt$$

$\Rightarrow \ln|I| = -\frac{R}{L}t + \ln C$

$\Rightarrow \ln|I| - \ln C = -\frac{R}{L}t$

$\Rightarrow \ln\left|\frac{I}{C}\right| = -\frac{R}{L}t$

$\Rightarrow \frac{I}{C} = e^{-\frac{R}{L}t}$

$\Rightarrow I = Ce^{-\frac{R}{L}t}$ ——— (1)

$I(0) = I_0$ i.e $I = I_0$ when $t = 0$

Putting in equation (1)

$$I_0 = Ce^{-\frac{R}{L}\cdot 0}$$

$$\Rightarrow \boxed{C = I_0}$$

Putting the value of C in equation (1)

$$\boxed{I = I_0 e^{-\frac{R}{L}t}}$$

MEDHOD OF SUBSTITUTION

Take the complex term in the diffrential equation as z. Then find out $\frac{dz}{dx}$.

put the value of z and $\frac{dy}{dx}$ in the given diffrential equation.

Then solve it by variable & separable find out the value of z.

Finally put the value of z.

NO- 10, 20, 21, 22, 25

10. $y' = (y+4x)^2$ ——— (1)

put $z = y + 4x$

Diff. both sides w.r. to x

$$\frac{dz}{dx} = \frac{dy}{dx} + 4 \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 4$$

Putting in equation (1)

$$\frac{dz}{dx} - 4 = z^2$$

$$\Rightarrow \frac{dz}{dx} = z^2 + 4$$

$$\Rightarrow \frac{dz}{z^2+4} = dx$$

Integrating both sides.

$$\int \frac{dz}{z^2+4} = \int dx$$

$$\Rightarrow \frac{1}{2} \tan^{-1}\left(\frac{z}{2}\right) = x + c$$

$$\Rightarrow \frac{1}{2} \tan^{-1}\left(\frac{y+4x}{2}\right) = x + c.$$

22. $xy' = y + x^2 \sec\left(\frac{y}{x}\right)$ $\quad y(1) = \pi$ ——(1)

Put $\frac{y}{x} = z$

$$\Rightarrow \frac{xy' - y.1}{x^2} = \frac{dz}{dx}$$

$$\Rightarrow xy' - y = x^2 \frac{dz}{dx}$$

Putting in equation (1)

$$xy' - y = x^2 \sec\left(\frac{y}{x}\right)$$

$$\Rightarrow x^2 \frac{dz}{dx} = x^2 \sec z$$

$$\Rightarrow \frac{dz}{dx} = \frac{1}{\cos z}$$

$$\Rightarrow \cos z \, dz = dx$$

Integrating both sides,

$$\int \cos z \, dz = \int dx$$

$$\Rightarrow \sin z = x + c$$

$$\Rightarrow \sin\left(\frac{y}{x}\right) = x + c \quad \text{——(2)}$$

$y(1) = \pi$ i.e $y = \pi$ when $x = 1$

Putting in equation (2)

$$\sin\left(\frac{\pi}{1}\right) = 1 + c$$

$$\Rightarrow 0 = 1 + c \Rightarrow \boxed{c = -1}$$

Putting the value of c in equation (2) the solution is $\sin\left(\frac{y}{x}\right) = x - 1$

25. $$y' = \frac{1 - 2y - 4x}{1 + y + 2x} \quad \text{---- (1)}$$

Put $y + 2x = z$

Diff. both sides w.r to x

$$\frac{dy}{dx} + 2 = \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 2$$

Putting in equation (1)

$$\frac{dz}{dx} - 2 = \frac{1 - 2z}{1 + z}$$

$$\Rightarrow \frac{dz}{dx} = \frac{1 - 2z}{1 + z} + 2$$

$$\Rightarrow \frac{dz}{dx} = \frac{3}{1 + z}$$

$$\Rightarrow (1 + z)\, dz = 3\, dx$$

Integrating both sides.

$$\int (1 + z)\, dz = \int 3\, dx$$

$$\Rightarrow z + \frac{z^2}{2} = 3x + c$$

$$\Rightarrow (y + 2x) + \frac{(y + 2x)^2}{2} = 3x + c.$$